

## Appendix A

## APPENDIX A CALCULATIONS FOR ESTIMATING RANDOM ERROR

The purpose of this analysis is to provide guidelines for estimating the maximum probable random error ( $\pm 3\sigma$ ) involved in taking a single measurement in the field or in deriving a single point source test result. Each probable random error is based on the expected precision with which a piece of equipment can be read.

Any systematic errors due to improper calibration, faulty equipment, or inadequate test procedures will be in addition to the probable random error.

In this analysis, expected random errors for direct measurements are given first. Then the compounded random errors expected in calculated results are shown. The following statistical method is used to compute random error.

If  $Z = f(x, x_2 \dots)$  and is differentiable, then

$$e_z = \sqrt{\sum \left( \frac{\partial z}{\partial x_i} \right)^2 e_i^2}$$

$e_z$  = probable maximum error in  $z$   
 $e_i$  = probable maximum error in  $x_i$

### EXAMPLE

$$V_o = C \frac{V_m P_b}{T_m}$$

where:

$V_o$  = corrected meter volume  
 $V_m$  = uncorrected meter volume  
 $P_b$  = barometric pressure  
 $T_m$  = meter temperature  
 $C$  = a constant

$$\frac{\delta V_o}{\delta V_m} = C \frac{P_b}{T_m}$$

$$\frac{\delta V_o}{\delta P_b} = C \frac{V_m}{T_m}$$

$$\frac{\delta V_o}{\delta T_m} = C \frac{V_m P_b}{T_m^2}$$

Let:  $e_{V_o}$  = The probable maximum error in  $V_o$   
 $e_{V_m}$  = The probable maximum error in  $V_m$   
 $e_{P_b}$  = The probable maximum error in  $P_b$   
 $e_{T_m}$  = The probable maximum error in  $T_m$

Then:

$$e_{V_o}^2 = \left( C \frac{P_b}{T_m} \right)^2 e_{V_m}^2 + \left( C \frac{V_m}{T_m} \right)^2 e_{P_b}^2 + \left( -C \frac{V_m P_b}{T_m^2} \right)^2 e_{T_m}^2$$

Divide by  $V_o^2$ :

$$\frac{e_{V_o}^2}{V_o^2} = \frac{\frac{C^2}{T_m^2} P_b^2 e_{V_m}^2 + V_m^2 e_{P_b}^2 + \frac{V_m^2 P_b^2}{T_m^2} e_{T_m}^2}{\frac{C^2 V_m^2 P_b^2}{T_m^2}}$$

Let:

$$\gamma_{V_o} = \frac{e_{V_o}}{V_o} = \text{The relative error in } V_o$$

$$\gamma_{V_m} = \frac{e_{V_m}}{V_m} = \text{The relative error in } V_m$$

$$\gamma_{P_b} = \frac{e_{P_b}}{P_b} = \text{The relative error in } P_b$$

$$\gamma_{T_m} = \frac{e_{T_m}}{T_m} = \text{The relative error in } T_m$$

Then:

$$\gamma_{V_o} = [\gamma^2_{V_m} + \gamma^2_{P_b} + \gamma^2_{T_m}]^{1/2}$$

### NUMERICAL EXAMPLE

The data used in this example were taken from an actual particulate source test. The maximum expected random error for each measurement is presented in the following format:

$$\pm \gamma_i = \frac{e_i}{x_i} (100)$$

where:  $\gamma_i$  = relative uncertainty, %  
 $e_i$  = uncertainty of measurement  
 $x_i$  = actual reading from equipment

1. Velocity head, P

$$\begin{aligned}\gamma P &= \frac{0.03 \text{ in H}_2\text{O}}{0.44 \text{ in H}_2\text{O}} \times 100 \\ &= \pm 0.68\%\end{aligned}$$

\*includes fluctuations in reading  
due to turbulence in the stack

2. Duct gauge pressure, P<sub>D</sub>

$$\begin{aligned}\gamma P_D &= \frac{0.03 \text{ in Hg}}{1.00 \text{ in Hg}} \times 100 \\ &= \pm 0.3\%\end{aligned}$$

3. Stack temperature, T<sub>D</sub>

$$\begin{aligned}\gamma T_b &= \frac{5^\circ \text{R}}{1130^\circ \text{R}} \times 100 \\ &= \pm 0.4\%\end{aligned}$$

4. Meter temperature, T<sub>m</sub>

$$\begin{aligned}\gamma T_m &= \frac{5^\circ \text{R}}{514^\circ \text{R}} \times 100 \\ &= \pm 0.97\% \cong 1.0\%\end{aligned}$$

5. Meter volume, V<sub>m</sub>

$$\begin{aligned}\gamma V_m &= **2 \times \frac{0.02 \text{ ft}^3}{25 \text{ ft}^3} \times 100 \\ &= 0.16\%\end{aligned}$$

\*\*accounts for initial  
and final meter readings

6. Saturated gas temperature, T<sub>s</sub>

$$\begin{aligned}\gamma T_s &= \frac{0.5^\circ \text{R}}{498^\circ \text{R}} \times 100 \\ &= \pm 0.1\%\end{aligned}$$

7. Saturated gas pressure in impingers, P<sub>s</sub>

This is based on the saturated gas temperature in the impingers which is accurate to  $\pm 1/2^\circ \text{F}$ .

$$\begin{aligned}\gamma P_s &= \frac{P_{38^\circ \text{F}} - P_{37.5^\circ \text{F}}}{P_{38^\circ \text{F}}} \\ \gamma P_s &= \frac{0.2292 \text{ "Hg} - 0.2248 \text{ "Hg}}{0.2292 \text{ "Hg}} \times 100 = \pm 1.9\%\end{aligned}$$

8. Condensate weight,  $W_c$

$$\begin{aligned}\gamma W_c &= *** 2 \times \frac{0.05 \text{ gm}}{120 \text{ gm}} \times 100 \\ &= \pm 0.1\%\end{aligned}$$

\*\*\*accounts for two weighings

9. Filter weights,  $W_p$

$$\begin{aligned}\gamma W_p &= *** 2 \times \frac{0.002 \text{ gm}}{0.10 \text{ gm}} \times 100 \\ &= \pm 0.4\%\end{aligned}$$

10. Pitot tube coefficient,  $c_p$  estimated as  $\pm 1.2\%$  by the EPA.

11. Stack diameter,  $D$

$$\begin{aligned}\gamma D &= \frac{1 \text{ in}}{168 \text{ ins}} \times 100 \\ &= \pm 0.60\%\end{aligned}$$

12. Time,  $t$

$$\begin{aligned}\gamma t &= \frac{0.01 \text{ min}}{60 \text{ mins}} \times 100 \\ &= \pm 0.02\%\end{aligned}$$

13. Nozzle diameter,  $D_n$

$$\begin{aligned}\gamma D_n &= \frac{0.1 \text{ mm}}{6 \text{ mm}} \times 100 \\ &= \pm 1.7\%\end{aligned}$$

14. Barometric pressure,  $P_b$

$$\begin{aligned}\gamma P_b &= \frac{0.05'' \text{ Hg}}{30'' \text{ Hg}} \times 100 \\ &= \pm 0.2\%\end{aligned}$$

## UNCERTAINTY ERRORS IN CALCULATED RESULTS

Corrected meter volume,  $V_o$

$$V_o = \frac{P_b V_m}{T_m}$$

$$\gamma^2 V_o = \gamma^2 V_m + \gamma^2 P_b + \gamma^2 T_m$$

$$\begin{aligned}\gamma V_o &= [(0.16)^2 + (0.2)^2 + (1.0)^2]^{1/2} \\ &= 1.03\%\end{aligned}$$

Water vapor content

$$\%H_2O = \frac{0.0474 W_c + C}{V_o + 0.0474 W_c + C}$$

where:

$$C = \frac{V_o P_s}{P_b - P_i - P_s}$$

$$\gamma^2 C = \gamma^2 V_o + 2\gamma^2 P_s + \gamma^2 P_b + \gamma^2 P_i$$

$$\gamma C = [(1.9)^2 + 2(1.9)^2 + (0.2)^2 + (2.9)^2]^{1/2}$$

$$\gamma C = \pm 4.4\%$$

$$\gamma^2 H_2O = 2\gamma^2 W_c + 2\gamma^2 C + \gamma^2 V_o$$

$$\begin{aligned}\gamma H_2O &= [2(0.1)^2 + 2(4.4)^2 + (1.03)^2]^{1/2} \\ &= \pm 6.3\%\end{aligned}$$

Assume:  $\%H_2O = 2 - eH_2O = \pm 1.3\%$  water vapor

The error in the dry portion (1 -  $\%H_2O$ ) is:

$$\begin{aligned}\gamma_{dry} &= \gamma(1 - H_2O) = \frac{1.3}{80} \times 100 \\ &= \pm 1.6\%\end{aligned}$$

Molecular weight, MW

The Orsat Analyzer is estimated to be accurate within  $\pm 0.2\%$  for each component.

$$\gamma^2 CO_2 = \gamma_{dry}^2 + \gamma^2 CO_2 = (1.6)^2 + \left(\frac{0.2}{8}\right)^2 \cong (1.6)^2$$

$$\gamma^2 O_2 = \gamma_{dry}^2 + \gamma^2 O_2 = (1.6)^2 + \left(\frac{0.2}{8}\right)^2 \cong (1.6)^2$$

$$\gamma^2 \text{CO} = \gamma^2 \text{dry} + \gamma^2 \text{CO} = (1.6)^2 + \left(\frac{0.2}{1}\right)^2 \cong (1.6)^2$$

$$\gamma^2 \text{N}_2 = \gamma^2 \text{dry} + \gamma^2 \text{N}_2 = (1.6)^2 + \left(\frac{0.2}{85}\right)^2 \cong (1.6)^2$$

$$= \gamma^2 \text{CO}_2 + \gamma^2 \text{CO}_2 + \gamma^2 \text{CO} + \gamma^2 \text{N}_2$$

$$\begin{aligned} \gamma \text{MW} &= [4(1.6)^2]^{1/2} \\ &= \pm 3.2\% \end{aligned}$$

Stack gas velocity

$$V_s = 85.55 C_p [\Delta p^{1/2}] \text{ avg} \left[ \frac{T_D}{(P_D + P_b) \text{ MW}} \right]^{1/2}$$

$$\gamma^2 V_s = \gamma^2 C_p + 0.25(\gamma \Delta P)^2 + 0.25[\gamma^2 T_D + \gamma^2 P_d + \gamma^2 \text{MW}]$$

$$\gamma V_s = [(1.2)^2 + 0.25(6.8)^2 + 0.25\{(0.4)^2 + (0.2)^2 + (3.0)^2 + (3.2)^2\}]^{1/2}$$

$$\gamma V_s = \pm 4.2\%$$

Stack cross-sectional area

$$A = \frac{\pi D^2}{4}$$

$$\gamma^2 A = 1/2 (\gamma_D)^2$$

$$\gamma A = 1/2 (0.60)^2$$

$$\gamma A = \pm 0.30$$

Stack gas flow rate

$$Q_o = \frac{(10.6) V_s A (P_b + P_d)(100 - \% \text{H}_2\text{O})}{T_D}$$

$$\gamma^2 Q_o = \gamma^2 V_s + \gamma^2 P_b + \gamma^2 \text{dry} + \gamma^2 T_D$$

$$\gamma Q_o = [(4.23)^2 + (0.30)^2 + (0.2)^2 + (3.0)^2 + (1.6)^2 + (0.4)^2]^{1/2}$$

$$\gamma Q_o = \pm 5.5\%$$

### Particulate grain loading

$$G = \frac{15.43W}{V_o}$$

$$\gamma^2 G = \gamma^2 W + \gamma^2 V_o$$

$$\gamma G = [(4.0)^2 + (1.03)^2]^{1/2}$$

$$\gamma G = \pm 4.13\%$$

### Particulate mass emission rate

$$M = \frac{Q_o G}{116.7}$$

$$\gamma^2 M = \gamma^2 G + \gamma^2 Q_o$$

$$\gamma M = [(4.13)^2 + (5.5)^2]^{1/2}$$

$$\gamma M = \pm 6.9\%$$

### Isokinetic ratio I

$$I = \frac{(100 - \%H_2O) AV_s t T_m}{(T_D)(Q_m)}$$

$$\gamma^2 I = \gamma^2 \text{dry} + \gamma^2 A + \gamma^2 V_s + \gamma^2 t + \gamma^2 T_D + \gamma^2 V_m$$

$$\gamma I = [(1.6)^2 + (0.3)^2 + (4.2)^2 + (0.02)^2 + (1.0)^2 + (0.4)^2 + (.16)^2]^{1/2}$$

$$\gamma I = \pm 4.6\%$$



## SUMMARY

<u>Measured Quantities</u>	<u><math>\pm</math> Relative Uncertainty, %</u>
Velocity Head, P	0.68
Duct Gage Pressure, $P_D$	0.3
Stack Temperature, $T_D$	0.4
Meter Temperature, $T_m$	1.0
Meter Volume, $V_m$	0.2
Saturated Gas Temperature, $T_s$	0.1
Saturated Gas Pressure, $P_s$	1.9
Condensate Weight, $W_c$	0.1
Filter Weights, $W_p$	0.4
Pitot Tube Coefficient, $C_p$	1.2
Stack Diameter, D	0.6
Time, t	0.02
Nozzle Diameter, $D_n$	1.7
Barometric Pressure, $P_b$	0.2
<u>Calculated Quantities</u>	
Corrected Meter Volume, $V_o$	1.03
Water Vapor Content, % $H_2O$	6.3
Dry Volume, (1 - % $H_2O$ )	1.6
Molecular Weight, MW	3.2
Stack Gas Velocity, $V_s$	4.2
Stack Cross-Sectional Area, A	0.3

Stack Gas Flow Rate, $Q_0$	5.5
Particulate Grain Loading, G	4.13
Particulate Mass Emission Rate, M	6.9
Isokinetic Ratio, I	4.6